REPLACEMENT PROBLEM

By

Yogendra Mani Tripathi – Asst. Professor

Faculty of Management and Technology, Harish Chandra PG College

LECTURE CONTENT

- Introduction to Replacement Problem
- Types or Classification of Replacement Problem
- Simple Numerical based on types of Replacement problem

Replacement Problem – Introduction

- In Industries, all equipments are put to continuous use which reduces the efficiency of the equipment. The study of replacement is concerned with situations that arise when some items such as machines, electric-light bulbs, etc., need replacement due to their deteriorating efficiency, failure or breakdown.

- The deteriorating efficiency or complete breakdown may be either gradual or all of a sudden. In all such situations, there is a need to formulate a most economic replacement policy for replacing faulty units or to take some remedial special action to restore the efficiency of deteriorating units.

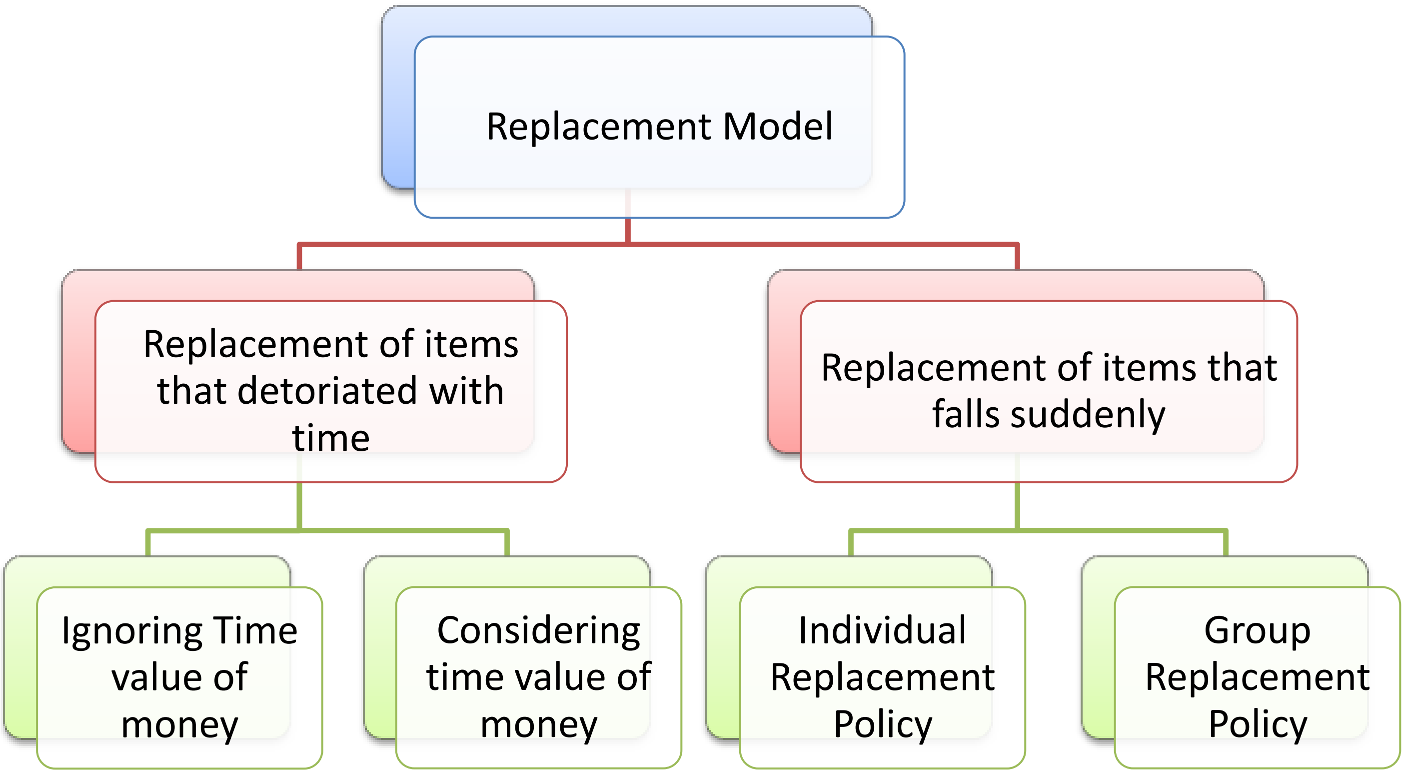
- A replacement is also needed for the equipment if the cost incurred in operating and maintaining the equipment exceeds the benefit derived out of it.

- The objective of the Replacement problem is therefore to determine the optimal time at which the equipment is to be replaced with new one.

Types of Replacement Problem

Replacement problems, in general, are of three types.

1. Replacement of items that deteriorate with time.
2. Replacement of items that break down completely, and
3. Replacement of items that becomes out of date due to new developments.



REPLACEMENT OF ITEMS THAT DETERIORATE WITH TIME

Generally, the maintenance cost of certain items, e.g., machine, always increases gradually with time and a stage comes when the maintenance cost becomes so large that it is better and economical to replace the item with a new one. There may be a number of alternative choices and in each choice, we make a comparison between various alternatives by considering and safety risks, etc.

REPLACEMENT OF ITEMS THAT FAIL COMPLETELY

We always come across practical situations in real life where the failure of certain item occurs all of a sudden, instead of gradual deterioration (e.g., the failure of an electric light-bulb). The failure of the item may result in complete breakdown of the system.

If the time of failure can be predicted, preventive replacement will often be the appropriate course of action. However, in many cases it may not be possible to predict failure time accurately. In such cases we shall assume that the probability distribution of failure time may be obtained, based on the past experience. Here it is assumed that the failure occurs only at the end of a certain period, say till time (t). The problem is to determine an optimal value of ***'t'*** so as to minimize the total cost involved in the system.

We shall consider the following two types of replacement policies:

1. Individual replacement policy: Under this policy, an item is replaced immediately after its failure.
2. Group replacement policy: Under this policy, we take decision as to when all the items must be replaced, irrespective of the fact that items have failed or have not failed, with a provision that if any item fails before the optimal time, it may be individually replaced.

Numerical based upon Replacement Problem of items that deteriorates with time

Case – 1 : Replacement Policy when Value of Money does not change with time

The objective here is to determine the optimum replacement age of an equipment/ item whose running/maintenance cost increases with time and the value of money remains static during that period.

Solution: *An optimum replacement policy suggest that replace the equipment at the end of n years,* ***if the average total cost in the (n+1)th year is more than the average total cost in the nth year and the nth year's maintenance cost is less than the previous year's average total cost.***

Let C = Capital Cost of Equipment, S= scrap value of equipment

n = number of years that equipment would be in use

f (t) =maintenance cost function and, A(n) = Average total annual cost.

Question 1: A firm is considering replacement of a machine, whose cost price is Rs. 12,200 and the scrap value is Rs. 200. The running (maintenance and operating) cost are found from experience are as follows:

Year	1	2	3	4	5	6	7	8
Running Cost	200	500	800	1200	1800	2500	3200	4000

When should the machine be replaced?

Solution:

In the problem given above we are provided with running cost f(t), Scrap Value S=200 and the cost of the machine C = Rs. 12,200.

In order to find out the optimal time n when the machine would be replaced, we will calculate the average total cost per year during the life of the machine as shown in the table below:

Years (1)	**Running Cost f(n) (2)**	**Cumulative running cost ∑ f(n) (3)**	**Depreciation Cost = C-S (4)**	**Total Cost TC = (3) + (4) (5)**	**Average Total Cost A(n)= (5)/(1) (6)**
1	200	200	12000	12,200	12,200
2	500	700	12000	12,700	6,350
3	800	1500	12000	13,500	4,500
4	1200	2700	12000	14,700	3,675
5	1800	4500	12000	16,500	3,300
6	2500	7000	12000	19,000	**3,167**
7	3200	10200	12000	22,200	3,171
8	4000	14200	12000	26,200	3,275

From the above table we can see that the average total cost A(n) is minimum at the end of 6^{th} year and then from 7^{th} year onwards it starts increasing, hence we would take a decision to replace the machine at the end of the 6^{th} year where the value of A(n) is minimum.

Question 2: The data collected in running a machine, the cost of which is Rs. 60,000 are given below

Year	1	2	3	4	5
Resale Value	42000	30000	20400	14400	9650
Cost of Spares	4000	4270	4880	5700	6800
Cost of Labour	14000	16000	18000	21000	25000

Determine the optimum period for the replacement of the machine.

Solution: To determine the running cost we add the cost of spares and cost of labour.

Years (1)	**Running Cost f(n) (2)**	**Cumulative running cost ∑ f(n) (3)**	**Resale Value (4)**	**Depreciation Cost = C-S (5)**	**Total Cost TC = (5) + (3) (6)**	**Average Total Cost A(n)= (6)/(1) (7)**
1	18000	18000	42000	18,000	36,000	36,000
2	20270	38270	30000	30,000	68,270	34,135
3	22880	61150	20400	39,600	1,00,750	33,583
4	26700	87850	14400	45,600	1,33,450	**33,362**
5	31800	1,19,650	9650	50,350	1,70,000	34,000

From the above table we can see that the average total cost A(n) is minimum at the end of 4th year and then from 5th year onwards it starts increasing, hence we would take a decision to replace the machine at the end of the 4th year where the value of A(n) is minimum i.e Rs. 33,362/-.

Numerical based upon Replacement Problem of items that fails suddenly.

It is usually very difficult to predict the time when particular equipment will fail suddenly. This problem can be overcome by determining the probability distribution of failures. Also, it is presumed that the failure occurs only at the end of the period say *t*.

Thus the objective is to find the value of *t* which minimizes the total cost involved for the replacement.

In such situation, there are two types of replacement policies are being followed:

(a) Individual Replacement Policy : Under this policy, an item is replaced immediately upon its failure.

(b) Group Replacement Policy: Under this policy, it is decided to replace all the items after a certain time period irrespective of the facts that items have failed or have not failed with an option that if any item before the optimal time, it may be individually replaced.

Question 3: The following failures have been observed for a certain type of transistors in a digital computer:

End of Week	1	2	3	4	5	6	7	8
Probability of failure to date	.05	.13	.25	.43	.68	.88	.96	1.00

Total number of transistors at the beginning of assembly is 1000 units. The cost of replacing the individual failed transistors is Rs. 1.25/-. The decision is made to replace all these transistors simultaneously at fixed intervals and to replace the individual transistors as they fails in service. If the cost of group replacement is 30 paisa per transistors, what is the best interval between group replacement?

Solution:

Let Pi be the probability that a transistors which was new when placed in position for use, fails during the ith week of its life. Thus we have,

P1 = 0.05
P2 = 0.13 – 0.05 = 0.08
P3 = 0.25 – 0.13 = 0.12
P4 = 0.43 – 0.25 = 0.18
P5 = 0.68 – 0.43 = 0.25
P6 = 0.88 – 0.68 = 0.20
P7 = 0.96 – 0.88 = 0.08
P8 = 1.00 – 0.96 = 0.04

Let Ni denotes the number of replacement made at the end of ith week. Then we have,

N0 = Number of transistors at the beginning = 1000

N1 = N0P1 = 1000 * 0.05 = 50

N2= N0P2 + N1P0 = 1000*0.08 + 50*0.05 = 82

N3= N0P3 + N1P2 + N2P1 = 1000*0.12 + 50*0.08 + 82*0.05 = 128

N4= N0P4 + N1P3 + N2P2 + N3P1 = 199

N5= N0P5 + N1P4 + N2P3 + N3P2 + N4P1 = 289

N6 = N0P6 + N1P5 + N2P4 + N3P3 + N4P2 + N5P1 = 272

N7 = N0P7 + N1P6 + N2P5 + N3P4 + N4P3 + N5P2 + N6P1 = 194

N8 = N0P8 + N1P7 + N2P6 + N3P5 + N4P4 + N5P3 + N6P2 + N7P1 = 195

Now, as we know that group replacement of all the 1000 transistors at one go cost 30 paisa per transistors and the replacement of individual transistors on failure cost Rs.1.25, the average cost for different group replacement policies are given as under:

End of Week	Individual Replacement	Total Cost (Individual + Group Replacement)	Average Cost
1	50	50*1.25 + 1000 x 0.30 = 363	363
2	50+82=132	132*1.25 + 1000*0.30 = 465	232.50
3	132+128=260	260*1.25 + 1000 * 0.30 = 625	**208.30**
4	260+199= 459	459 * 1.25 + 1000 * 0.30 = 874	218.50
5	459+289= 748		
6	748+272= 1020		
7	1020+194= 1214		
8	1214+195 = 1409		

Since the average cost is lowest at week 3 hence the optimum interval between group replacement is **3 weeks.**